HORNSBY GIRLS HIGH SCHOOL

2011
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading Time- 5 minutes
- Working Time – 3 hours
- Write using a black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question in a new booklet.

Total marks (120)

- Attempt Questions 1– 10
- All questions are of equal value

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Total Marks – 120

Attempt Questions 1-10
All Questions are of equal value

Begin each question in a NEW writing booklet, writing your student number and question number in the boxes indicated. Extra writing booklets are available.

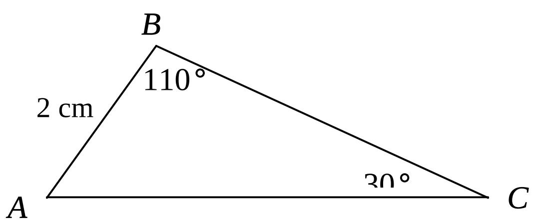
Question 1 (12 marks) Use a SEPARATE booklet. **Marks**

(a) Find $3^{3.5}$ correct to three significant figures. **2**

(b) Factorise completely: $3x^2 - 12y^2$ **2**

(c) Simplify $\dfrac{3}{m+2} - \dfrac{1}{m}$. **2**

(d) Calculate the length of side *BC*, giving your answer to one decimal place. **2**



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(e) Solve $|2x-3| < 11$ and graph the solution on a number line. **2**

(f) Find the limiting sum of the geometric series $1 + \dfrac{3}{4} + \dfrac{9}{16} + \dfrac{27}{64} + ...$ **2**

Question 2 (12 marks) Use a SEPARATE booklet. **Marks**

(a) Differentiate (i) $x \sin\left(\frac{\pi}{4} - x\right)$ **2**

(ii) $\frac{2x-3}{\tan x}$ **2**

(b) (i) Find $\int \frac{\sqrt{x}}{x^5}\, dx$ **2**

(ii) Evaluate $\int_0^4 \frac{dx}{3x+1}$ **3**

(c) Find the equation of the tangent to the curve $y = e^{x^3}$ at the point whose x-coordinate is 2. **3**

Question 3 (12 marks) Use a SEPARATE booklet. **Marks**

(a) The point $Q(-2,1)$ lies on the line k whose equation is $9x-2y+20=0$.
The point $R(4,-2)$ lies on the line l whose equation is $3x+y-10=0$.

(i) Show that the lines k and l intersect at the point $P(0,10)$. **2**

(ii) Show that the equation of the line m which joins Q and R is **2**

$$x+2y=0$$

(iii) Find, as a surd, the perpendicular distance from P to m. **2**

(iv) Hence, or otherwise, find the exact value of the area of the triangle bounded by the three lines k, l and m. **2**

(b) The first three terms of an arithmetic series are 50, 43 and 36.

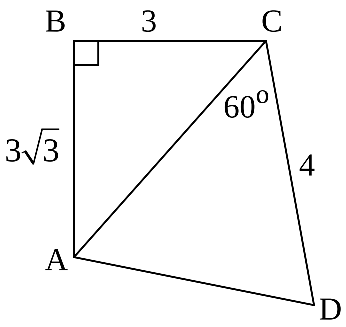
(i) Write down the n^{th} term of the series. **1**

(ii) If the last term of the series is -13, how many terms are there in the series? **2**

(iii) Find the sum of the series. **1**

Question 4 (12 marks) Use a SEPARATE booklet. **Marks**

(a) In the figure ABCD, AB = $3\sqrt{3}$, BC = 3, CD = 4, $\angle ABC = 90^{\circ}$ and $\angle ACD = 60^{\circ}$.
All length measurements are in metres.



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Find (i) the length of AC. **1**

(ii) the exact length of AD. **2**

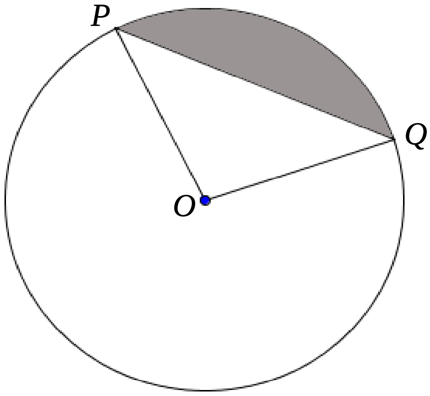
(iii) the exact area of the figure ABCD. **2**

(b) Prove that $(\cot\theta + \operatorname{cosec}\theta)^2 = \dfrac{1+\cos\theta}{1-\cos\theta}$. **2**

(c) The curve $y = \sec 2x$, for $0 \le x \le \dfrac{\pi}{6}$, is rotated about the x axis. **3**

Find the volume of the solid of revolution generated.

(d) Find the area of the minor segment shaded, given $\angle QOP = 0.6$ radians and the radius of the circle, centre O, is 4 metres. **2**



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Question 5 (12 marks) Use a SEPARATE booklet. **Marks**

(a) Let $\log_a 2 = x$ and $\log_a 3 = y$. Find an expression for $\log_a 12$, in terms of x and y. **2**

(b) A function $f(x)$ is given by $f(x) = \begin{cases} x+6, & for \;\; x \le 3 \\ x^2 - 9, & for \;\; x > 3 \end{cases}$ **2**

Find $f(3) - f(5)$.

(c) Solve the equation $3^{2x} + 2(3^x) - 15 = 0$. **2**

(d) Consider the function $f(x) = 1 - 3x + x^3$, for the domain $-2 \le x \le 3$.

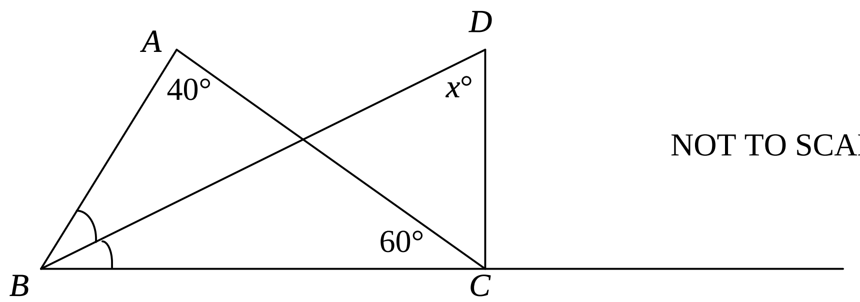
(i) There are two turning points for $y = f(x)$. Find their co-ordinates and determine their nature. **3**

(ii) Draw a sketch of the curve $y = f(x)$ for the domain $-2 \le x \le 3$, clearly showing the turning points, *y*-intercept and the endpoints. **3**

Question 6 (12 marks) Use a SEPARATE booklet. **Marks**

(a) In the diagram below (not drawn to scale), BD bisects $\angle ABC$, DC is perpendicular to BC, $\angle BAC = 40°$, $\angle ACB = 60°$ and $\angle BDC = x°$. **3**

Copy the diagram into your writing booklet.
Find the value of x, giving reasons for each step in your calculation.



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(b) The mass, M in grams, of a radioactive substance may be expressed as

$M = 120e^{-0.04t}$ where t is the time in years

(i) What was the initial mass of the radioactive substance? **1**

(ii) Find the mass of the substance after 10 years. **1**

(iii) Find the instantaneous rate of change of the mass after 10 years. **2**

(iv) After how many years will the mass of the substance be 15 grams? **2**

(c) Let $f(x) = x^3 - 6x^2 + kx + 4$, where k is a constant. **3**
Find the values of k for which $f(x)$ is an increasing function.

Question 7 (12 marks) Use a SEPARATE booklet. **Marks**

(a) Let α and β be the roots of $2x^2 - 9x + 2 = 0$.

(i) Find $\alpha\beta$. **1**

(ii) Hence find $\beta + \frac{1}{\beta}$ **1**

(b) Two functions are defined as $f(x) = \sin 2x$ and $g(x) = \sin x$.
It is known that $\sin 2x = 2\sin x \cos x$ for all values of x. (Do not show this)

(i) The equation $f(x) = g(x)$ has solutions $x = 0$ and $x = \pi$. **2**
Find the third solution in the domain $0 \le x \le \pi$.

(ii) Sketch $y = f(x)$ and $y = g(x)$ on the same set of axes in the domain **2**
$0 \le x \le \pi$, showing the intercepts of both curves.

(iii) Find the area enclosed between $y = f(x)$ and $y = g(x)$ between **3**
$x = 0$ and $x = \pi$.

(c) Find the equation of the locus of a point $P(x, y)$ that moves so that its distance **3**
from the point $(-2, 4)$ is equal to its distance from the line $y = 6$.

Question 8 (12 marks) Use a SEPARATE booklet. **Marks**

(a) At time t seconds, the position x cm of a particle moving in the straight line $X'OX$ is given by $x = at^2 + bt$ cm, where a and b are constants.
The particle initially passes through the origin, O, with velocity 16 cm/s in the positive direction and, after 8 seconds, the particle is again at O.

(i) Find the velocity of the particle at any time, in terms of a and b. **1**

(ii) Find the values of the constants a and b. **2**

(iii) When AND where is the particle at rest? **2**

(iv) With reference to acceleration and displacement, describe the motion of the particle. **2**

(b) Elise invests \$50 000 in an account which earns 8% interest, compounded annually. She intends to withdraw \$$M$ at the end of each year, immediately after the interest has been paid. She wishes to be able to do this for exactly 20 years, so that the account will then be empty.

(i) Write an expression for the amount of money she has in the account immediately after she has made her first withdrawal? **1**

(ii) Show that the amount of money in the account, immediately after her 20^{th} withdrawal is: **2**

$$\$50000 \times 1.08^{20} - \$M(1 + 1.08 + 1.08^2 + ... + 1.08^{19})$$

(iii) Calculate the value of M which leaves her account empty after the 20^{th} withdrawal. **2**

Question 9 (12 marks) Use a SEPARATE booklet. **Marks**

(a) If $y = e^{2x}$, show that $\dfrac{d^2y}{dx^2} = 2y + \dfrac{dy}{dx}$. **2**

(b) The volume of a crop to be harvested, changes at the rate of $\dfrac{dV}{dt}$ cubic metres per week, where $\dfrac{dV}{dt} = \dfrac{1400}{(7t+1)^2}$, and t is the time in weeks since the harvest was started.

(i) Find the volume of the crop as a function of t. **1**

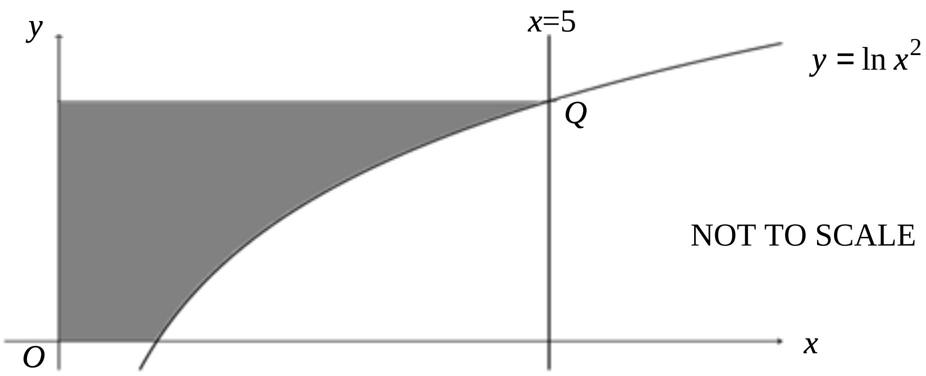
(ii) Initially, the volume of the crop to be harvested was calculated to be 1600 cubic metres. Find the volume after one week. **2**

(iii) At what exact time is the volume changing at half the initial rate? **2**

(c) (i) Show that $\dfrac{d}{dx}(x\ln x - x) = \ln x$. **2**

(ii) Hence, or otherwise, find $\int \ln x^2 \, dx$. **1**

(iii) The graph shows the curve $y = \ln x^2$, $(x > 0)$ which meets the line $x = 5$ at Q. Using your answers from (i) and (ii), or otherwise, find the area of the shaded region. **2**



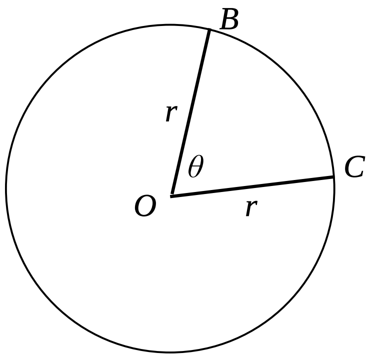
Question 10 (12 marks) Use a SEPARATE booklet. **Marks**

(a) Use Simpson's rule with 3 function values to find an approximate value of **2**

$$\int_{3.7}^{4.3} \frac{1}{1+\sqrt{x}}\, dx\ .$$

Give your answer correct to two decimal places.

(b) *OBC* is a sector of a circle with centre *O*. *OB* and *OC* are radii of length r metres, of the circle. The arc *BC* of the circle subtends an angle θ radians at *O*.
The perimeter of the sector is 12 metres.



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(i) Show that the area, A, of the sector *OBC* is given by **2**

$$A = \frac{72\theta}{(\theta+2)^2}\ .$$

(ii) Hence, or otherwise, find the maximum area of the sector. **3**

(c) A poster is being designed to have an area of 324 cm^2. The poster is to be framed in a rectangular frame. The frame is made of timber which has a width of 3 cm at the bottom and on each side and a width of 5 cm along the top.

(i) If the rectangular frame has an outer width of x cm and an outer length of y cm, show that the area, A cm^2, of the poster and frame is given by $$A = x\left[8 + \frac{324}{x-6}\right]$$ **2**

(ii) Find the values of x and y such that the outer perimeter of the frame is as short as possible. **3**

End of paper

HGHS 2Unit TRIAL - 2011

Q1

a) $3^{3.5} = 46.7653\ldots$

$= 46.8$ (3 sig. fig.)

b) $3x^2 - 12y^2$

$= 3(x^2 - 4y^2)$

$= 3(x+2y)(x-2y)$

c) $\frac{3}{m+2} - \frac{1}{m}$

$= \frac{3m - (m+2)}{m(m+2)}$

$= \frac{3m - m - 2}{m(m+2)}$

$= \frac{2m-2}{m(m+2)}$

$= \frac{2(m-1)}{m(m+2)}$

d) $\angle A = 180° - 110° - 30°$

$= 40°$

$\frac{BC}{\sin 40°} = \frac{2}{\sin 30°}$

$BC = \frac{2\sin 40°}{\sin 30°}$

$= 2.57\ldots$

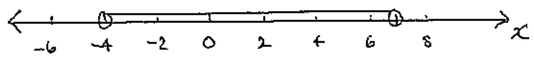
$= 2.6$ (1 d.p)

e) $|2x-3| < 11$

$-11 < 2x - 3 < 11$

$-8 < 2x < 14$

$-4 < x < 7$



f) $S_\infty = \frac{a}{1-r}$, $a = 1$ $r = \frac{3}{4}$

$= \frac{1}{1 - \frac{3}{4}}$

$= \frac{1}{\frac{1}{4}}$

$= 4$

Q2 a) (i) $\frac{d}{dx} x \sin\left(\frac{\pi}{4} - x\right) = -x\cos\left(\frac{\pi}{4} - x\right) + \sin\left(\frac{\pi}{4} - x\right)$

$= \sin\left(\frac{\pi}{4} - x\right) - x\cos\left(\frac{\pi}{4} - x\right)$

(ii) $\frac{(\tan x)\cdot 2 - (2x-3)\sec^2 x}{\tan^2 x} = \frac{2\tan x - (2x-3)\sec^2 x}{\tan^2 x}$

b) (i) $\int x^{\frac{1}{2} - 5}\, dx = \int x^{-\frac{9}{2}}\, dx$

$= \frac{-2x^{-\frac{7}{2}}}{7} + C$

$= -\frac{2}{7} \cdot \frac{1}{x^3\sqrt{x}} + c$ or $\frac{-2\sqrt{x}}{7x^4} + c$

(ii) $\frac{1}{3}\left[\ln(3x+1)\right]_0^4 = \frac{1}{3}\ln(13) - \ln 1$

$= \frac{1}{3}\ln 13$

c) $\frac{dy}{dx} = 3x^2 e^{x^3}$ when $x = 2$, $y = e^8$

$\frac{dy}{dx} = 3 \times 4e^8$
$= 12e^8$

Eqn of tangent given by $\frac{y - e^8}{x - 2} = 12e^8$

$y - e^8 = 12e^8(x-2)$

$y = 12e^8 x - 24e^8 + e^8$

$y = 12e^8 x - 23e^8$

QUESTION 3

a) i) $9x - 2y + 20 = 0$ — ①
$3x + y - 10 = 0$ — ②
② × 2 ⇒ $6x + 2y - 20 = 0$ — ③
① + ③ ⇒ $15x = 0$
$x = 0$
subst $x = 0$ into ①
$-2y + 20 = 0$
$2y = 20$
$y = 10$
∴ $P(0, 10)$

ii) $\frac{y-1}{x+2} = \frac{-2-1}{4+2}$
$\frac{y-1}{x+2} = \frac{-1}{2}$
$2y - 2 = -x - 2$
$x + 2y = 0$

iii) $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$
$= \frac{|1 \times 0 + 2 \times 10 + 0|}{\sqrt{1+2^2}}$
$= \frac{20}{\sqrt{5}}$
$= 4\sqrt{5}$

iv) $QR = \sqrt{(-2-4)^2 + (1+2)^2}$
$= \sqrt{36+9}$
$= \sqrt{45}$
$= 3\sqrt{5}$

v) $A = \frac{3\sqrt{5} \times 4\sqrt{5}}{2} = 30.$

b) i) $T_n = a + (n-1)d.$
$= 50 + (n-1) \times -7$
$= 50 - 7n + 7$
$= 57 - 7n.$

ii) $-13 = 57 - 7n$
$7n = 70$
$n = 10$

iii) $S_n = \frac{n}{2}(2a + (n-1)d)$
$S_{10} = \frac{10}{2}(2 \times 50 + (9 \times -7))$
$= 5(100 - 63)$
$= 5 \times 37$
$= 185.$

Q4 a) (i) $AC^2 = (3\sqrt{3})^2 + 3^2$ (Pythagoras' theorem)
$= 36$
$AC = 6$ (length absolute).

(ii) $AD^2 = 6^2 - 4^2 - 2 \times 6 \times 4 \cos 60°$
$= 28$
$AD = \sqrt{28}$
$= 2\sqrt{7}$

(iii) Area of figure $= \frac{1}{2} \times 3 \times 3\sqrt{3} + \frac{1}{2} \times 6 \times 4 \sin 60°$
$= \frac{9\sqrt{3}}{2} + \frac{12\sqrt{3}}{2}$
$= \frac{21\sqrt{3}}{2}.$

b) LHS $= \cot^2\theta + 2\cot\theta\operatorname{cosec}\theta + \operatorname{cosec}^2\theta$
$= \frac{\cos^2\theta}{\sin^2\theta} + \frac{2\cos\theta}{\sin\theta} \times \frac{1}{\sin\theta} + \frac{1}{\sin^2\theta}$
$= \frac{\cos^2\theta + 2\cos\theta + 1}{\sin^2\theta}$
$= \frac{(\cos\theta + 1)^2}{1 - \cos^2\theta}$
$= \frac{(\cos\theta+1)(\cos\theta+1)}{(\cos\theta+1)(1-\cos\theta)}$
$=$ RHS.

OR LHS $= \left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)^2$
$= \frac{(\cos\theta + 1)^2}{\sin^2\theta}$
$= \frac{(\cos\theta + 1)^2}{1 - \cos^2\theta}$
$= \frac{(\cos\theta+1)(\cos\theta+1)}{(1-\cos\theta)(1+\cos\theta)}$
$=$ RHS.

c) $V = \pi\int_0^{\frac{\pi}{6}} \sec^2 2x\, dx$
$= \pi\left[\frac{1}{2}\tan 2x\right]_0^{\frac{\pi}{6}}$
$= \frac{\pi}{2}\left(\tan\frac{\pi}{3} - \tan 0\right)$
$= \frac{\pi}{2}(\sqrt{3} - 0)$
$= \frac{\pi\sqrt{3}}{2}$ units³

d) $A = \frac{1}{2}r^2\theta^c - \frac{1}{2}r^2\sin\theta^c$
$= \frac{1}{2} \times 4^2 \times 0.6 - \frac{1}{2} \times 4^2 \times \sin 0.6$
$\doteqdot 0.282860\ldots$ units²

Question 5.

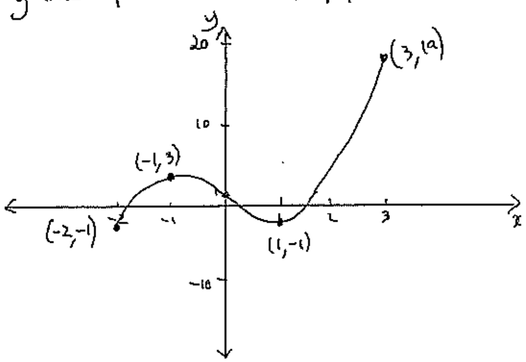
a) $\log_a 12 = \log_a(3\times 2^2)$
$= \log_a 3 + \log_a 2^2$
$= \log_a 3 + 2\log_a 2$
$= y + 2x$
$= 2x + y$

b) $f(3) - f(5) = (3+6) - (5^2 - 9)$
$= 9 - 16$
$= -7$

c) $3^{2x} + 2(3^x) - 15 = 0$
$(3^x)^2 + 2(3^x) - 15 = 0$
Let $y = 3^x$
$\therefore y^2 + 2y - 15 = 0$
$(y+5)(y-3) = 0$
$y = 3$ or -5
ie $3^x = 3$ or -5
So $3^x = 3$ or $3^x = -5$
$x = 1$ no solution.

d) i) $f(x) = 1 - 3x + x^3$
$f'(x) = -3 + 3x^2$
$0 = -3 + 3x^2$
$3x^2 = 3$ $f''(x) = 6x$
$x^2 = 1$
$x = \pm 1$
when $x = 1$, $y = 1 - 3 + 1 = -1$ $f''(1) = 6 > 0$
$\therefore$ Min turning pt at $(1, -1)$
When $x = -1$, $y = 1 + 3 - 1 = 3$ $f''(-1) = -6 < 0$
$\therefore$ Max turning pt at $(-1, 3)$

ii) When $x = -2$, $f(-2) = 1 - 3(-2) + (-2)^3 = -1$
When $x = 3$, $f(3) = 1 - 3(3) + 3^3 = 19$
y-intercept is 1.



QUESTION 6

a) $\angle ABC + 40 + 60 = 180$ ($\angle$ sum, $\triangle ABC$)
$\angle ABC = 80$
$\angle DBC = 40$ (DB bisects $\angle ABC$)
$x + 90 + 40 = 180$ ($\angle$ sum $\triangle BDC$)
$x = 50$

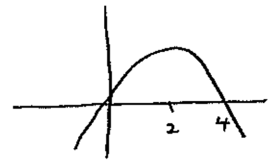
b) i) 120
ii) $M = 120e^{-0.04\times 10}$
$= 120e^{-0.4}$
$= 80.438$
$= 80$ g.

iii) $\frac{dM}{dt} = 120 \times -0.04 e^{-0.04t}$
$= -4.8e^{-0.04t}$
when $t = 10$.
$\frac{dM}{dt} = -4.8e^{-0.4}$
$= -3.2175$
$= -3.2$ g/year.

iv) $15 = 120e^{-0.04t}$
$e^{-0.04t} = \frac{15}{120}$
$-0.04t = \ln(0.125)$
$t = \frac{\ln(0.125)}{-0.04}$
$= 51.98$
$= 52$ years.

c) $f'(x) = 3x^2 - 12x + k \geq 0$.
~~$x = \frac{12 \pm \sqrt{144 - 12k}}{6}$~~
$\Delta < 0$
$b^2 - 4ac \leq 0$
$144 - 12k \leq 0$
$12k > 144$
$k \geq 12$.

OR $3x^2 - 12x + k \geq 0$
$3x^2 - 12x \geq -k$
$k > 12x - 3x^2$
$k \geq 3x(4 - x)$



when $x = 2$
$k > 6 \times 2$
$k > 12$

Question 7 Solutions

(a) $2x^2 - 9x + 2 = 0$

(i) $\alpha\beta = \frac{2}{2}$

$= 1$

(ii) From (i)

$\alpha = \frac{1}{\beta}$

$\beta + \frac{1}{\beta} = \beta + \alpha$

$= \frac{9}{2}$

(b)(i) $f(x) = g(x)$

$\sin 2x = \sin x$

$2\sin x \cos x = \sin x$

$2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0$

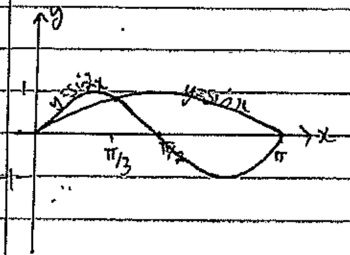
$x = 0, \pi$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$

(ii)



$A_1 = \int_0^{\pi/3} (\sin 2x - \sin x)\,dx$

$= \left[-\frac{1}{2}\cos 2x + \cos x\right]_0^{\pi/3}$

$= \left(\cos\frac{\pi}{3} - \frac{1}{2}\cos\frac{2\pi}{3}\right) - \left(-\frac{1}{2}\cos 0 + \cos 0\right)$

$= \left(\frac{1}{2} - \frac{1}{2} \times -\frac{1}{2}\right) - \left(-\frac{1}{2} + 1\right)$

$= \frac{1}{2} + \frac{1}{4} - \frac{1}{2}$

$= \frac{1}{4}$ units^2

$A_2 = \int_{\pi/3}^{\pi} (\sin x - \sin 2x)\,dx$

$= \left[-\cos x + \frac{1}{2}\cos 2x\right]_{\pi/3}^{\pi}$

$= \left(-\cos\pi + \frac{1}{2}\cos 2\pi\right) - \left(-\cos\frac{\pi}{3} + \frac{1}{2}\cos\frac{2\pi}{3}\right)$

$= \left(1 + \frac{1}{2}\right) - \left(-\frac{1}{2} - \frac{1}{4}\right)$

$= 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{4}$

$= \frac{9}{4}$

$\therefore A = \frac{10}{4}$ units^2

$= \frac{5}{2}$ units^2

(c) $P = (x, y)$ $A = (-2, 4)$ $M = (x, 6)$

$PA = PM$

$PA^2 = PM^2$

$(x+2)^2 + (y-4)^2 = (y-6)^2$

$x^2 + 4x + 4 + y^2 - 8y + 16 = y^2 - 12y + 36$

$x^2 + 4x + 20 = y^2 - y^2 - 12y + 8y + 36$

$x^2 + 4x + 20 = -4y + 36$

$x^2 + 4x - 16 = -4y$

$y = -\frac{x^2}{4} - x + 4$

Locus is $y = -\frac{x^2}{4} - x + 4$

Question 8.

a) i) $x = at^2 + bt$

$v = \frac{dx}{dt}$

$= 2at + b$

ii) When $t = 0$, $v = 16$:

$\therefore 16 = 2a(0) + b$

$b = 16$

When $t = 8$, $x = 0$:

$0 = a(8^2) + 16(8)$

$= 64a + 128$

$a = -2$

$\therefore a = -2$ and $b = 16$.

iii) at rest when $v = 0$

ie $0 = 2at + b$

$= -4t + 16$

$4t = 16$

$t = 4$

$\therefore$ at rest in 4 seconds

when $t = 4$, $x = -2t^2 + 16t$

$= -2(16) + 16(4)$

$= 32$ cm

$\therefore$ particle is 32 cm from the origin when it is at rest.

iv) $a = 2a$

$= -4$.

Acceleration is always negative. Particle moves through the origin and away from it for 4 seconds till it is 32 cm away. It then turns round and heads back to the origin. Will meet the origin in 8 seconds + will continue on.

Particle is slowing down till it turns, then it speeds up towards the origin.

b) i) $A_1 = \$50\,000(1+0.08)^1 - \$M$

$= \$50\,000(1.08) - \$M$.

ii) After 2 years,

$A_2 = A_1(1.08) - M$

$= (50000(1.08) - M) \times 1.08 - M$

$= 50000(1.08)^2 - 1.08M - M$

$= 50\,000(1.08)^2 - M(1.08 + 1)$

After 3 years,

$A_3 = A_2(1.08) - M$

$= (50000(1.08)^2 - 1.08M - M) \times 1.08 - M$

$= 50\,000(1.08)^3 - 1.08^2M - 1.08M -$

$= 50000(1.08)^3 - M(1.08^2 + 1.08 + 1)$

After 20 years

$A_{20} = A_{19}(1.08) - M$

$= 50000(1.08)^{20} - M(1.08^{19} + 1.08^{18} + \ldots + 1.08^2 + 1.08 + 1)$

$= 50000(1.08)^{20} - M(1 + 1.08 + \ldots + 1.08^{19})$

iii) For $(1 + 1.08 + \ldots + 1.08^{19})$, $a = 1$, $r = 1.08$, $n = 20$.

$S_{20} = \frac{1(1.08^{20} - 1)}{0.08}$

Account is empty when $A = 0$

So $0 = 50000(1.08)^{20} - M\left(\frac{1.08^{20} - 1}{0.08}\right)$

$M\left(\frac{1.08^{20} - 1}{0.08}\right) = 50000(1.08)^{20}$

$M = \frac{50\,000(1.08)^{20}}{\frac{1.08^{20} - 1}{0.08}}$

$= \$5092.61$

Q9 a) $y = e^{2x}$

$\frac{dy}{dx} = 2e^{2x}$

$\frac{d^2y}{dx^2} = 4e^{2x}$

RHS $= 2y + \frac{dy}{dx}$

$= 2(e^{2x}) + 2e^{2x}$

$= 4e^{2x}$

$= \frac{d^2y}{dx^2}$

$=$ LHS.

b) $\frac{dV}{dt} = \frac{1400}{(7t+1)^2}$

(i) $V = \int 1400\,(7t+1)^{-2}\,dt$

$= 1400\,\frac{(7t+1)^{-1}}{-1\times 7} + c$

$= \frac{-200}{7t+1} + c$

$\left.\begin{matrix} t=0 \\ V=1600 \end{matrix}\right\}$ $1600 = \frac{-200}{1} + c$

$c = 1800$

$V = 1800 - \frac{200}{7t+1}$

$t=1$ $\quad V = 1800 - \frac{200}{8}$

$= 1775$

$t=0,\ \frac{dV}{dt} = 1400$

$\frac{1}{2}$(initial rate) (half) $= 700$

$700 = \frac{1400}{(7t+1)^2}$

$\frac{1}{2} = \frac{1}{(7t+1)^2}$

$(7t+1)^2 = 2$

$7t+1 = \pm\sqrt{2}$

$t = \frac{-1\pm\sqrt{2}}{7}$

But $t>0$, $\therefore t = \frac{\sqrt{2}-1}{7}$

c) (i) $\frac{d}{dx}(x\ln x - x) = x\times\frac{1}{x} + (\ln x)\times 1 - 1$

$= 1 + \ln x - 1$

$= \ln x$

(ii) $\int \ln x^2\,dx = 2\int \ln x\,.dx$

$= 2(x\ln x - x) + C$

(iii) Q is $(5, \ln 25)$, x intercept of $y = \ln x^2$

$2\ln x = 0$ or $\therefore x^2 = 1$

$\therefore x = 1$

$A = 5\ln 25 - \int_0^5 \ln x^2\,dx$

$= 5\ln 25 - 2\left[x\ln x - x\right]_1^5$

$= 5\ln 25 - 2\left[5\ln 5 - 5 - (0-1)\right]$

$= 5\ln 5^2 - 10\ln 5 + 10 - 2$

$= 8$ units^2

QUESTION 10

a) $\int_{3\cdot 7}^{4\cdot 3} \frac{1}{1+\sqrt{x}}\,dx = \frac{4\cdot 3 - 3\cdot 7}{6}\left[0\cdot 342 + 4\times 0\cdot 333 + 0\cdot 325\right]$

$= 0\cdot 20$

b) i) $A = \frac{1}{2}r^2\theta$ —— (1)

$P = r\theta + 2r = 12$

$12 = r(\theta+2)$

$r = \frac{12}{\theta+2}$ —— (2)

$\therefore A = \frac{\theta}{2}\times\frac{12^2}{(\theta+2)^2}$

$= \frac{144\theta}{2(\theta+2)^2}$

$= \frac{72\theta}{(\theta+2)^2}$

ii) $\frac{dA}{d\theta} = \frac{(\theta+2)^2.72 - 144\theta(\theta+2)}{(\theta+2)^4}$

$= \frac{72(\theta+2)\left[\theta+2-2\theta\right]}{(\theta+2)^4}$

$= \frac{72\left[2-\theta\right]}{(\theta+2)^3}$

Max Area when $\frac{dA}{d\theta} = 0$

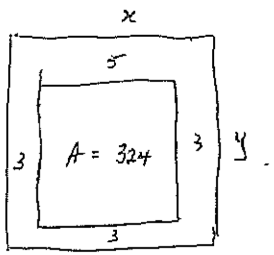
i.e. $2-\theta = 0$

$\theta = 2$.

$\therefore$ Max Area $= \frac{72\times 2}{(2+2)^2}$

$= 9$.

c)



i) $(x-6)(y-8) = 324.$

$$y - 8 = \frac{324}{x-6}$$

$$y = \frac{324}{x-6} + 8$$

$$A = xy$$

$$= x\left(8 + \frac{324}{x-6}\right)$$

ii) $P = 2x + 2y.$

$$= 2x + 2\left(\frac{324}{x-6} + 8\right)$$

$$= 2x + 628(x-6)^{-1} + 16.$$

$$\frac{dP}{dx} = 2 - 628(x-6)^{-2}$$

$$= 2 - \frac{628}{(x-6)^2}$$

min when $\frac{dP}{dx} = 0.$

i.e. $628 = 2(x-6)^2.$

$324 = (x-6)^2$

$x - 6 = \pm 18$

$x = -12, 24.$

$\therefore x = 24$

$$y = \frac{324}{24-6} + 8$$

$$= 26.$$